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CONCATENATION METHOD FOR HIGH-TEMPORAL RESOLUTION SSVEP-BCI

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Abstract: Electroencephalographic (EEG) signals are generally non-stationary, however, nearly stationary brain responses, such as steady-state visually evoked potentials (SSVEP), can be recorded in response to repetitive stimuli. Although Fourier transform has precise resolution with long time windows (5 or 10 s for instance) to extract SSVEP response (1-100 Hz ranges), its resolution with shorter windows decreases due to the Heisenberg-Gabor uncertainty principle. Therefore, it is not easy to extract evoked responses such as SSVEP within short EEG epochs. This limits the information transfer rate of SSVEP-based brain-computer interfaces. In order to circumvent this limitation, we concatenate EEG signals recorded simultaneously from different channels, and we Fourier analyze the resulting sequence. From this constructed signal, high frequency resolution can be obtained with time epochs as small as only 1 s, which improves SSVEPs classification. This method may be effective for high-speed brain computer interfaces (BCI).

1 INTRODUCTION

Brain computer interfaces (BCI) are alternative methods to the normal outputs of the brain via the nerve-muscle system (Birbaumer, 2006). The purpose of BCI is to detect physiological signals from the brain and translate them into a control signal for an external device. It has been developed with surface electroencephalograms (EEG), electrocorticograms (ECoG), and implanted electrodes. Among them, surface EEG has many advantages: it is non-invasive, technically less demanding, and evoked responses are fast (within the millisecond range). Especially, short response times of EEG could enable users to control an external device almost in real-time (Sanei and Chambers, 2007; Bashashati et al., 2007; Lotte et al., 2007). However, nowadays the available BCI systems are generally constrained to execute commands using epochs of more than 3 s. For instance, in BCI word processing systems, it takes more than 3 s to type each letter. Therefore, being able to detect BCI commands with shorter time epochs (about 1 s) is a crucial problem.

EEG signals are known to be non-stationary[1] (see for instance (Kawabata, 1973)). However, in steady-state visually evoked potentials (SSVEP), EEG features at the stimulation frequency and its harmonics are nearly stationary (Vialatte et al., 2010). When subjects focus attention on flickering lights with constant frequencies, steady-state brain activity appears, predominantly in the occipital cortex, and propagates to other brain areas. These responses are better observed in the frequency, or time-frequency domains. Therefore, to detect these features, frequency or time-frequency analysis methods are applied, such as classical Fourier transform or wavelet transform (Quiroga et al., 2001; Vialatte et al., 2008; Bin et al., 2009). Moreover there are other methods such as empirical mode decomposition for instantaneous frequency (Huang et al., 1998).

To detect these SSVEPs with high temporal resolution, one generally uses short EEG epochs to

[1]Their statistical properties evolve with time

compute Fourier transforms. However, due to the Heisenberg-Gabor uncertainty principle, high frequency resolution cannot be obtained by Fourier transforming short signals: the shorter the window length, the lower the frequency resolution. Furthermore, when using short epochs, EEG becomes nearly stationary. Therefore for short epochs the Fourier representation of SSVEP and non-SSVEP activity are of comparable amplitudes, hence SSVEP peak is difficult to be detected. As a result, SSVEP cannot be detected reliably with short windows (a minimum of 3 s is usually required). To circumvent this limitation, we propose a concatenation method: EEG epochs measured simultaneously from different channels are concatenated, in order to generate an artificially longer epoch that can be analyzed with a better frequency resolution. The details of this method are described in the following sections. We then demonstrate this method on a real SSVEP classification task.

The organization of the paper is as follows. Section 2 details the procedure of the concatenation method. In section 3, an EEG electrode placement and the experimental procedure are presented. Results, discussions and conclusions are in sections 4, 5, and 6, respectively.

2 CONCATENATION METHOD

In this section, the concatenation method is detailed. Firstly, the non-stationarity of EEG is shown with different sizes of time epoch. Secondly, we propose the concatenation method to circumvent this problem.

2.1 Non-stationarity of EEG

EEGs are considered non-stationary. When using frequency analysis methods such as the Fourier transform and the wavelet transform, rhythmic components of the EEG (such as the theta or the alpha wave) are extracted for decomposing a signal into a set of frequency components. However, the Fourier transform relies on the assumption that the analyzed signal is strictly stationary, otherwise, the resulting spectrum will make little physical sense. Therefore, when using long time epochs, these non-stationary components have limited impact on the Fourier spectrum. On the other hand, when using short time windows, EEG frequency components become nearly stationary, with higher resulting Fourier amplitudes.

SSVEPs are nearly stationary evoked responses, much more stable than classical EEG signals. They have very narrow-band responses, with precise spectral properties (an SSVEP response at 10 Hz exhibit

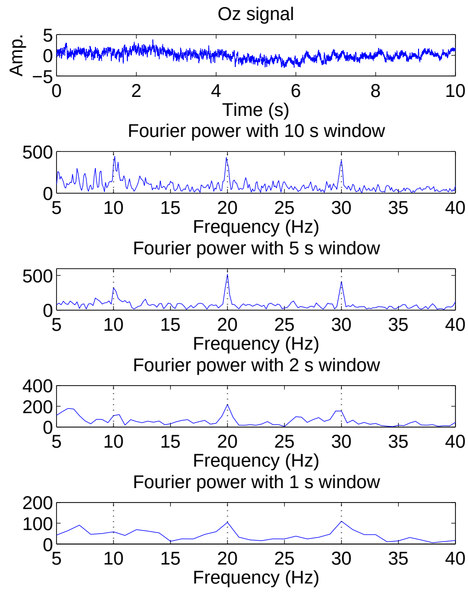


Figure 1: 10 Hz SSVEP responses: Top, Oz signal; Bottom four, Fourier power with different time windows of [0, *time*] (*time*∈[10, 5, 2, 1]). With the shorter windows, it shows lower time resolution and the peak is lower at the fundamental and harmonic frequencies.

a clear peak at 10 Hz, with a width below 1 Hz). These responses can be observed in a frequency domain shown in Fig. 1.

Frequencies of EEG signal were calculated by Fourier transform with different sizes of Hanning window function. The spectrum obtained with a 10 s time window exhibits sharp peak nears 10, 20, and 30 Hz. Conversely, the spectrum obtained with a 1 s time window exhibits lower peaks at these frequencies, and it has a lower frequency resolution. Notably, Fourier powers at frequencies surrounding stimulus frequency and its harmonics are higher when the time epoch is shorter. This is because, for long EEG epochs, the non-stationary components of EEG signals except the stimulus frequency have less impact on the Fourier spectrum. Thus, sharper and clearer SSVEP peaks can be obtained. On the other hand, for short EEG epochs, it has lower spectral amplitudes, which blurs out the SSVEP peak, which is furthermore distorted by the low frequency resolution.

2.2 Concepts of Concatenating

The concatenation method we propose is a way to improve artificially the frequency resolution while using very short EEG epochs. The details of this method are as follows.

Let $\mathbf{x_i}$ be the column vector of an signal observed in the ith channel. The concatenated signal $\mathbf{y}$ is con-

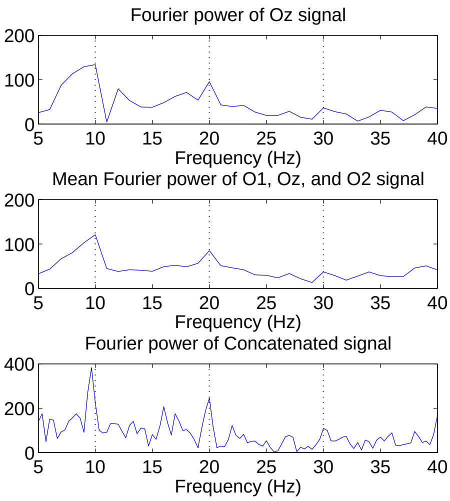


Figure 2: Frequency features of concatenated signal: Top, Fourier power $|X_{Oz}(e^{j\omega})|^2$; Middle, mean Fourier power of $|X_{cha}(e^{j\omega})|^2$ ($cha = [O1, Oz, O2]$); Bottom, Fourier power $|Y(e^{j\omega})|^2$ from a concatenated signal of O1, Oz, and O2.

structed by concatenating signals in the time domain:

$$\mathbf{y} = [\mathbf{x_1}^T, \mathbf{x_2}^T, \ldots, \mathbf{x_M}^T]^T, \tag{1}$$

where M is the number of concatenated signals and T represents transposition. If each signal is of length N, then $\mathbf{y}$ is of length MN. Furthermore, let $X_i(e^{j\omega})$ (resp $Y(e^{j\omega})$) be the Fourier transform of $\mathbf{x_i}$ (resp. $\mathbf{y}$). Frequency resolution of $Y(e^{j\omega})$ is M times higher than that of $X_i(e^{j\omega})$. It can be explained by the larger size of $\mathbf{y}$. Using $\omega = 2\pi f$ (f is frequency), short time Fourier transforms of these signals are expressed as

$$X_i(e^{j\omega}) = X_i(e^{2\pi jf}) = \sum_{t=0}^{N-1} \mathbf{x_i}[t] e^{-2\pi jft} \tag{2}$$

and

$$Y(e^{j\omega}) = Y(e^{2\pi jf}) = \sum_{t=0}^{MN-1} \mathbf{y}[t] e^{-2\pi jft}, \tag{3}$$

where

$$f = \frac{kL}{MN}[Hz] \ \ (k \in \mathbb{N}), \tag{4}$$

and t and L represent time index and the time length of 1 s, respectively. k represents the number of cycles of a sinusoidal signal within the time window of the Fourier transform. $e^{j\omega}$ should be 0 at the beginning and the end of the epoch. Frequency resolutions therefore are divided by the number of concatenations M.

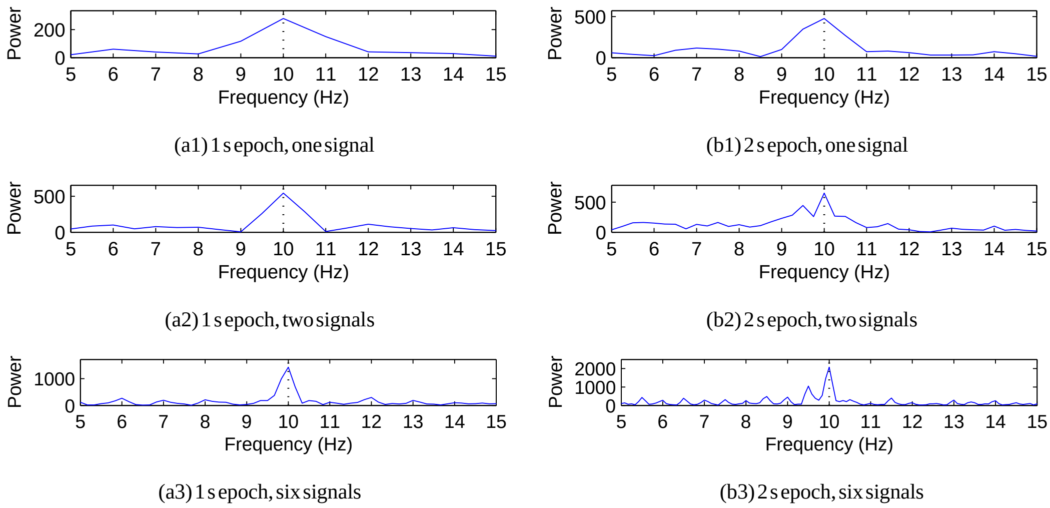
As an example, EEG electrodes are attached at Oz, O1, O2, Pz, P1, and P2 according to the international 10/20 system. SSVEP of all signals are measured while 10 Hz visual stimulus is displayed. $\mathbf{x_{Oz}}$, $\mathbf{x_{O1}}$, and $\mathbf{x_{O2}}$ correspond raw signals at Oz, O1, and O2, for instance. Concatenated signal $\mathbf{y}$ can be defined like $[\mathbf{x_{Oz}}^T, \mathbf{x_{O1}}^T, \mathbf{x_{O2}}^T]$. Here, we consider signals $\mathbf{x_{Oz}}$, and $\mathbf{y}$ ($[\mathbf{x_{Oz}}^T, \mathbf{x_{O1}}^T, \mathbf{x_{O2}}^T]$). Time length of each EEG signal $\mathbf{x_i}$ is 1 s (1 s epoch). In the Fig. 2, Fourier power $|X_{Oz}(e^{j\omega})|^2$ has weak peaks around 10 and 20 Hz. The mean of $|X_{cha}(e^{j\omega})|^2$ ($cha = [O1, Oz, O2]$) has similar peak of $|X_{Oz}(e^{j\omega})|^2$. Contrary to that, Fourier power of concatenated signal $|Y(e^{j\omega})|^2$ has stronger peak near 10 and 20 Hz.

Furthermore, we investigate effects of the number of signals: $\mathbf{x_{Oz}}$, $\mathbf{y_1}$ ($[\mathbf{x_{Oz}}^T, \mathbf{x_{O1}}^T]^T$), $\mathbf{y_2}$ ($[\mathbf{x_{Oz}}^T, \mathbf{x_{O1}}^T, \mathbf{x_{O2}}^T, \mathbf{x_{Pz}}^T, \mathbf{x_{P1}}^T, \mathbf{x_{P2}}^T]^T$). With these signals, Fig. 3 shows Fourier power $|X_{Oz}(e^{j\omega})|^2$, $|Y_1(e^{j\omega})|^2$, $|Y_2(e^{j\omega})|^2$ in two different conditions: $\mathbf{x_i}$ is 1 s or 2 s length. The Hanning window is of length MN. From the figure, regardless of 1 s epoch and 2 s epoch, the peak at 10 Hz appears more and more clearly as the number of concatenated signals increases. Thus, the concatenation method can extract sharper peak of SSVEP despite of short time epoch. Furthermore, different window functions can be used: no window function, Hanning window of same length as the concatenated signal, or Hanning window of same length as each signal, respectively. In Fig. 3, Hanning window of same length as the concatenated signal is applied and it is also used in the next sub-section.

Finally, concatenation of a EEG signals repetively were investigated. It uses only one EEG signal. This concatenated signal is expressed as $\mathbf{y_{same}} = [\mathbf{x}^T, \mathbf{x}^T, \ldots, \mathbf{x}^T]^T$. Fourier powers of this signal with different time epochs are shown in Fig. 4. These are small peaks at 1 Hz interval in Fig. 4-(a), 0.5 Hz intervals in Fig. 4-(b), and 0.2 Hz intervals in Fig. 4-(c). It may be a problem for detection of SSVEP peaks. It is obviously explained by the property of Fourier transform (see APPENDIX). Therefore, concatenation of EEG signals from different channels conduces better observation of SSVEP peaks.

3 EXPERIMENTAL DEMONSTRATION

Eight subjects took part in the experiment, and signed written informed consent forms. EEG signals were taken from a database recorded during SSVEP stimulation in Riken BSI/Japan. Photic stimulation is given using AVOTEC goggles and the flickering frequency is controlled by a shutter which allows a maximal refreshing rate of 293 Hz. A very broad range of frequencies is recorded (21 different frequencies from 1 Hz to 100 Hz): 1.00, 1.25, 1.88, 2.50, 3.33, 4.17,



(a1) 1 s epoch, one signal

(b1) 2 s epoch, one signal

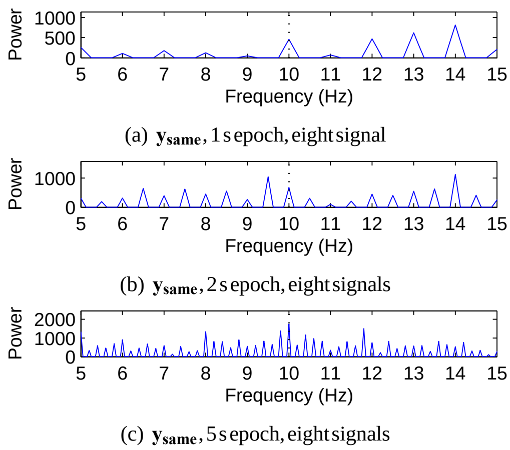
(a2) 1 s epoch, two signals

(b2) 2 s epoch, two signals

(a3) 1 s epoch, six signals

(b3) 2 s epoch, six signals

Figure 3: Frequency features of concatenated signal: Left three figures, length of each EEG signal is 1 s (1 s epoch); Right three figures, length of each EEG signal is 2 s (2 s epoch); Top two figures, Fourier power $|X_{Oz}(e^{j\omega})|^2$; Middle two figures, Fourier power $|Y_1(e^{j\omega})|^2$ ($\mathbf{y}_1$ is of Oz and O1); Bottom three figures, Fourier power $|Y_2(e^{j\omega})|^2$ ($\mathbf{y}_2$ is of Oz, O1, O2, Pz, P1, and P2).



(a) $\mathbf{y}_{\mathbf{same}}$, 1 s epoch, eight signal

(b) $\mathbf{y}_{\mathbf{same}}$, 2 s epoch, eight signals

(c) $\mathbf{y}_{\mathbf{same}}$, 5 s epoch, eight signals

Figure 4: Frequency features of $\mathbf{y}_{\mathbf{same}}$ with eight same signals: Top, 1 s epoch; Middle, 2 s epoch; Bottom, 5 s epoch.

5.00, 6.67, 8.33, 10.00, 13.33, 16.67, 20.00, 26.67, 33.33, 40.00, 53.33, 66.67, 80.00, 90.00, and 100.00 Hz. Firstly a subject sees a uniform grey screen for 20 s, then a flickering stimulus for 10 s, and then iteratively restarts this sequences of rest/stimulus condition. Stimuli were presented in a randomized order, in seven runs of nine frequencies, for a total of 63 trials (3 trials per 21 frequency).

EEG was recorded using a Biosemi system in a shielded room, with 128 active channels, all signals were amplified and digitized at 1024 Hz, after analog filtering of frequencies above 100 Hz and notch filtering at 50 Hz.

In this paper, parts of EEG channels are used as

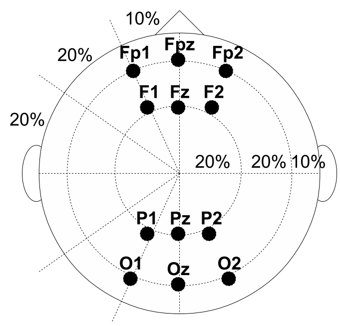


Figure 5: A placement of electrodes which are used for analysis in this paper.

shown in Fig. 5 (12 EEG channels: Fp1, Fpz, Fp2, F1, Fz, F2, P1, Pz, P2, O1, Oz, and O2) and visual stimuli of 10 and 13.33 Hz are used to investigate effectiveness of the concatenation method.

4 RESULTS

In this section, SSVEPs evoked by 10 and 13.33 Hz stimuli are investigated. First of all, EEG features to classify the SSVEPs are detailed. Then, SSVEPs of 10 and 13.33 Hz are classified with several features selected by a supervised feature ranking method, the Gram-Schmidt orthogonalization. Finally, we compare the frequency domain of a concatenated signal with that of the signals which are used for concatenation.

4.1 EEG Feature Extraction

SSVEP responses have peaks at the stimulus frequency and even harmonics (Fig. 1). Using 10 and 13.33 Hz as stimulus frequencies, SSVEPs are described by six different features for the purpose of classification: ξ^1, the spectral power at the stimulation frequency; ξ^2, the frequency power enhanced by SSVEP SNR (Vialatte et al., 2010) applied to the frequency domain; ξ^3, the magnitude square coherence function between two signals; ξ^4, global field synchronization; ξ^5 and ξ^6, the frequency power and SSVEP SNR of the concatenated signals.

The first feature is defined as

$$\xi^1(f) = |X(f)|^2 + |X(f\times 2)|^2, \quad (5)$$

i.e. sum of the spectral powers of the fundamental and its first harmonic. This feature is extracted at each channel of both 10 and 13.33Hz. Although ξ^2 has similar feature with ξ^1, SSVEP peaks can be enhanced by SSVEP SNR as

$$X'(f) = \frac{nX(f)}{\sum_{k=s}^{n/2} X(f+k\triangle f) + \sum_{k=s}^{n/2} X(f-k\triangle f)}, \quad (6)$$

and then the second feature can be defined as

$$\xi^2(f) = |X'(f)|^2 + |X'(f\times 2)|^2. \quad (7)$$

The coherence function, a synchrony measure, is distinguished into the magnitude square coherence function and the phase coherence function (see for instance (Nunez et al., 1997; Dauwels et al., 2010)). Magnitude square coherence function is the third feature we used. It is defined as

$$\xi^3_{i,j}(f) = \frac{|X_i(f)X_j^*(f)|^2}{|X_i(f)||X_j(f)|}, \quad (8)$$

where X^* is the complex conjugate of X, i and j correspond to the label of channel. It is a function of two signals, therefore, the number of pairs of all channels corresponds to the number of combinations.

Similarly, global field synchronization (GFS), another synchrony measure, is the fourth feature. GFS quantifies the synchrony of multiple signals. First of all, with Fourier transformed signals, one constructs the vectors $X_R(f) = (Re(X_1(f)), Re(X_2(f)), ..., Re(X_M(f)))^T$ and $X_I(f) = (Im(X_1(f)), Im(X_2(f)), ..., Im(X_M(f)))^T$, and computes the covariance matrix $C \in R^{2\times 2}$ for those two vectors. GFS ($\xi^4(f)$) is defined in terms of the normalized eigenvalues λ_1 and λ_2 of C:

$$\xi^4(f) = \lambda_1 - \lambda_2. \quad (9)$$

Finally, with concatenated signals, the Fourier power and SSVEP SNR are defined as

$$\xi^5(f) = |Y(f)|^2 + |Y(f\times 2)|^2, \quad (10)$$

$$\xi^6(f) = |Y'(f)|^2 + |Y'(f\times 2)|^2. \quad (11)$$

Here, we used six groups of channels for concatenation: (Fp1-Fpz-Fp2), (F1-Fz-F2), (P1-Pz-P2), (O1-Oz-O2), (Fp1-Fpz-Fp2-F1-Fz-F2), and (P1-Pz-P2-O1-Oz-O2).

There are six different features with different parameters. There are parameters of two stimulus frequencies and the channel locations for all features; and the window functions, and channel groups which are only for concatenated feature. We prepared two different conditions:

1. Condition 1 includes the Fourier, SSVEP SNR, coherence and GFS feature. In this condition, the total number of feature is $24(\xi^1) + 24(\xi^2) + 132(\xi^3) + 2(\xi^4) = 182$.
2. Condition 2 includes the Fourier and SSVEP SNR of each EEG signal, that of the concatenated signals, coherence and GFS feature. In this condition, the total number of features is $24(\xi^1) + 24(\xi^2) + 132(\xi^3) + 2(\xi^4) + 24(\xi^5) + 24(\xi^6) = 230$.

4.2 SSVEP Classification

For classifying the SSVEPs, the number of features must be as small as possible (see for instance (Dreyfus, 2005)). Input ranking thorough Gram-Schmidt orthogonalization is applied to select all relevant factors as inputs to the classifier, but only the relevant ones. The relevance is measured by the angle between the vector of a feature (ξ^i) and the vector of stimuli label (l) as

$$cos^2\theta_i = \frac{|(\xi^i)^T l|^2}{|(\xi^i)^T \xi||l^T l|}. \quad (12)$$

Firstly, a feature which is most correlated to the feature (l) is chosen by the largest $cos^2\theta_i$. l and all other candidate inputs are projected onto the null space of the selected input (subspace). The above procedure is iterated until all features have been ranked. With ranked features, SSVEPs are classified by linear discriminant analysis (LDA). Performances are defined as the classification accuracy (ACC) calculated with leave one out cross validation (LOOCV). These procedures are applied for two cases (two frequencies): using all features except concatenation features (Condition 1) and using all features (Condition 2). Additionally, we use four different time epochs (1.0, 2.0, 5.0, 10.0 s).

Table 1 shows ranked top 10 features thorough Gram-Schmidt orthogonalization. These results are performed with 1 s epoch. Mostly, ξ^1, ξ^2, ξ^3 with different parameters are chosen when concatenation

Table 1: Feature ranking with 1 s epochs: FT represents ξ^1, SNR represents ξ^2, MC represents ξ^3, ConcateFT represents ξ^5, and ConcateSNR represents ξ^6. The ranking is calculated firstly without the concatenation features (Condition 1), then including them (Condition 2). Concatenation features were chosen in high ranks.

Rank	Condition 1 (without councatenation)	Condition 2 (with councatenation)
1	FT (P22, 13.33 Hz)	ConcateFT (group6, 10 Hz, window2)
2	FT (F2, 10 Hz)	ConcateFT (group2, 13.33 Hz, window1)
3	SNR (Oz, 13.33 Hz)	ConcateSNR (group6, 10 Hz, window1)
4	SNR (O2, 10 Hz)	SNR (O2, 10 Hz)
5	MC (F1-P2, 10 Hz)	ConcateSNR (group3, 13.33 Hz, window3)
6	MC (F1-P2, 13.33 Hz)	ConcateSNR (group2-1, 13.33 Hz, window3)
7	MC (Fpz-P1, 10 Hz)	MC (F1-P2, 10 Hz)
8	MC (Fpz-F2, 13.33 Hz)	MC (Fpz-P1, 10 Hz)
9	MC (Fp2-Oz, 13.33 Hz)	MC (F1-O1, 10 Hz)
10	SNR (Fz, 13.33 Hz)	MC (Fp1-O2, 10 Hz)

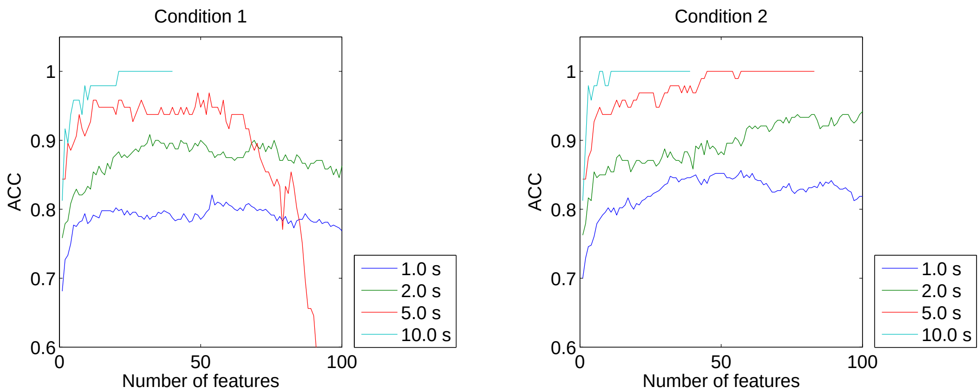


Figure 6: ACC result estimated by LDA. Left figure is in the condition without concatenation features. Right figure is in the condition with concatenation features. ACCs with 1 and 2 epoch look higher in Condition 2. ACC with 5 s is stable in Condition 2 although it seems to be over fitting to the training data in Condition 1 due to such as having too many features.

Table 2: ACC comparison between Condition 1 and Condition 2. Maximum ACC of all nu (($1 \leq nu \leq 100$)) is higher in Condition 2 in all time epochs. Furthermore, with 1 s epoch, ACC in Condition 2 is higher in all numbers of features and it shows $+0.06$ when $nu = 40$.

Features conditions		Number of features					Maximum of all	Minimum of all
		nu=1	nu=10	nu=20	nu=40	nu=100	nu ($1 \leq nu \leq 100$)	nu ($1 \leq nu \leq 100$)
t=1	Condition 1	0.68	0.78	0.80	0.79	0.77	0.82	0.68
	Condition 2	0.70	0.80	0.81	0.85	0.82	0.86	0.70
t=2	Condition 1	0.76	0.83	0.88	0.90	0.86	0.91	0.76
	Condition 2	0.76	0.86	0.87	0.86	0.94	0.94	0.76
t=5	Condition 1	0.84	0.92	0.94	0.95	–	0.97	0.59
	Condition 2	0.84	0.94	0.96	0.97	–	1.00	0.84
t=10	Condition 1	0.81	0.98	1.00	–	–	1.00	0.81
	Condition 2	0.81	0.98	1.00	–	–	1.00	0.81

features are not included. On the contrary, concatenation feature ξ^5 and ξ^6 are ranked top 6 only except rank 5 is ξ^2 when concatenating features are included. Furthermore, ACCs depending on number of features are shown in Fig. 6. Table 2 shows ACC shown in Fig. 6 when numbers of features are 1, 10, 20, 40, and 100. From the figure, ACCs with 1 and 2 epoch looks higher in Condition 2. Moreover, ACC with 5 s is stable in Condition 2 although it seems to be over fitting to the training data in Condition 1 due to such as having too many features. From the table, firstly, maximum ACC of all nu ($1 \leq nu \leq 100$) is higher in

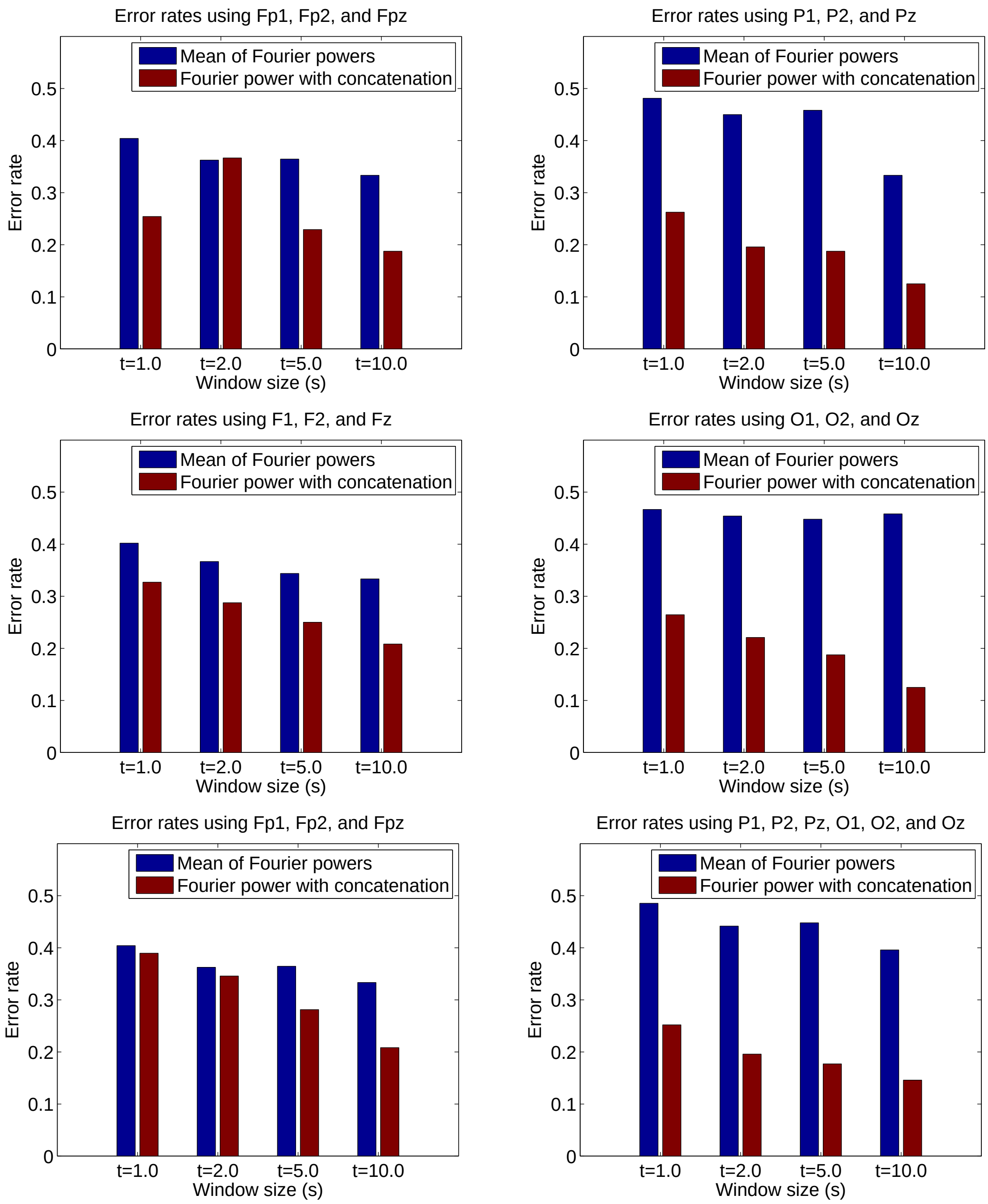


Figure 7: Training error rate comparing concatenation features and average of Fourier powers. In each figure, different EEG channels are used. Compared with these features, concatenation features has lower error rate than average of Fourier powers, especially in parietal and occipital areas.

Condition 2 in all time epochs. Secondly, with 1 s epoch, ACC in Condition 2 is higher in all numbers of features and it shows $+0.06$ when $nu = 40$.

4.3 Concatenation Method Vs. Averaging of Signals

Finally, we compare the frequency power of a concatenated signal with that of the signals which are used in concatenation. The average of Fourier powers of these signals is defined as

$$\xi^7(f) = \frac{1}{NumChannel} \sum_{channel}^{NumChannel} (\xi^1(f, channel)), \quad (13)$$

where *channel* and *NumChannel* represents each channel and total number of channels of each group, respectively. This feature is compared with the concatenation feature ξ^5 to prove the effectiveness of the concatenation method.

For comparison, SSVEP is classified with LDA classification with only ξ_5 or only ξ_7. There are two kinds of stimulus frequencies as parameters (10 and 13.33 Hz). Fig. 7 shows learning error rate in those cases. One can observe on this figure that error rates with concatenation features are systematically lower than those with averaged Fourier powers.

5 DISCUSSIONS

With the concatenation method, we can obtain not only higher frequency resolution but also higher peak of Fourier power at the stimulus frequency. Therefore, it may work effectively in the situation where high temporal resolution of SSVEP detection is required. This method only requires signals concatenation, consequently, it is not computationally demanding, and can be used in real time processing. It reduces the patients' stress when using SSVEP-BCI. The patients are not required to focus for more than one second, as evidenced by the 0.86 classification accuracy with 1 s epoch (Table 2).

In our future works, we intend to confirm effects of concatenating points with artificial signals, and in addition to EEG signals. Moreover, using a larger number of EEG channels could allow us to go down to even shorter EEG epochs. One of the unsolved problems is that each EEG epoch has different phase and amplitudes at their borders. Therefore, the cycles in each epoch do not match perfectly, which most probably reduces the SSVEP peaks in the Fourier spectrum. Improved algorithms for windowing EEG epochs, and for matching their phase differences, are to be developed.

6 CONCLUSIONS

A concatenation method is proposed to improve the frequency resolution of Fourier spectrum when using short EEG epochs. It successfully detected SSVEP response by concatenating EEG epochs in the time domain, down to 1 s windows. Classification tests on EEG data proved that concatenation method works better than the averaging of Fourier spectrums computed from the EEG epochs.

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APPENDIX

Since $\mathbf{y}$ is defined as (1), Fourier transform of $\mathbf{y}$ is expressed as

$$\begin{aligned} Y(e^{j\omega}) &= \sum_{t=0}^{MN-1} \mathbf{y}[t]e^{-j\omega t} \\ &= \sum_{t=0}^{N-1} \mathbf{x_1}[t]e^{-j\omega t} + \sum_{t=N}^{2N-1} \mathbf{x_2}[t-N]e^{-j\omega t} \\ &\quad + ... + \sum_{t=(M-1)N}^{MN-1} \mathbf{x_M}[t-(M-1)N]e^{-j\omega t} \\ &= \sum_{i=1}^{M} \sum_{t=(i-1)N}^{iN-1} \mathbf{x_i}[t-(i-1)N]e^{-j\omega t}. \end{aligned} \quad (14)$$

It also can be expressed as

$$Y(e^{2\pi jf}) = \sum_{i=1}^{M} \sum_{t=(i-1)N}^{iN-1} \mathbf{x_i}[t-(i-1)N]e^{-2\pi jft}, \quad (15)$$

using $\omega = 2\pi f$. Furthermore, if the same signals are concatenated (in the case of $\mathbf{y_{same}}$),

$$\mathbf{x_1} = \mathbf{x_2} = ... = \mathbf{x_M} = \mathbf{x}, \quad (16)$$

then, (15) becomes

$$Y(e^{2\pi jf}) = \sum_{i=1}^{M} \sum_{t=(i-1)N}^{iN-1} \mathbf{x}[t-(i-1)N]e^{-2\pi jft}. \quad (17)$$

Under this condition, the constant interval depending on the time epoch is in the case of

$$k = mM \ \ (m \in \mathbb{N}), \quad (18)$$

then, (19) becomes

$$f = \frac{kL}{MN} = \frac{mML}{MN} = \frac{mL}{N}. \quad (19)$$

In the case of this condition,

$$e^{-2\pi jft} = e^{-2\pi jf(t+nN)} \ \ (n = ... -2, -1, 0, 1, 2, ...) \quad (20)$$

then, (17) becomes

$$Y(e^{2\pi jf}) = M \sum_{t=0}^{N-1} \mathbf{x}[t]e^{-2\pi jft}. \quad (21)$$

Therefore, if (16) and (18), Fourier spectrum is M times higher than that of an EEG signal and it has same power distribution with each signal. Considering time epochs of 1 s, 2 s, and 5 s in Fig. 4, the emphasized frequencies are expressed as

$$f = \begin{cases} m & if \ L = N \, (1\, s\, epoch) \\ m/2 & if \ L = 2N \, (2\, s\, epoch) \\ m/5 & if \ L = 5N \, (5\, s\, epoch) \end{cases} \quad (22)$$

using (19).